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Contents

Abstract.
In a recent paper, Lubinsky proved eigenvalue distribution results for a class of Hankel matrix sequences arising in several applications, ranging from Padé approximation to orthogonal polynomials and complex analysis. The results appeared in Linear Algebra and its Applications, and indeed many of the statements, whose origin belongs to the field of approximation theory and complex analysis, contain deep results in (asymptotic) linear algebra. Here we make an analysis of a part of these findings by combining his derivation with previous results in asymptotic linear algebra, showing that the use of an already available machinery shortens considerably the considered part of the derivations. Remarks and few additional results are also provided, in the spirit of bridging (numerical and asymptotic) linear algebra results and those coming from analysis and pure approximation theory.

Key Words.
Hankel and Toeplitz matrix, matrix sequence, eigenvalue and singular value distribution, eigenvalue and singular value clustering

AMS Subject Classifications.
15B05, 15A18, 47B35

Abstract.
In the present paper, a Nyström-type method for second kind Volterra integral equations is introduced and studied. The method makes use of generalized Bernstein polynomials, defined for continuous functions and based on equally spaced points. Stability and convergence are studied in the space of continuous functions. Numerical tests illustrate the performance of the proposed approach.

Key Words.
Volterra integral equations, Nyström method, generalized Bernstein polynomials

AMS Subject Classifications.
41A10, 65R20, 65D32

Abstract.
There is extensive mathematical literature on the inverse problem of deautoconvolution for a function with support in the unit interval $[0, 1] \subset \mathbb{R}$, but little is known

about the multidimensional situation. This article tries to fill this gap with analytical and numerical studies on the reconstruction of a real function of two real variables over the unit square from observations of its autoconvolution on $[0,2]^2 \subset \mathbb{R}^2$ (full data case) or on $[0,1]^2$ (limited data case). In an L^2-setting, twofoldness and uniqueness assertions are proven for the deautoconvolution problem in 2D. Moreover, its ill-posedness is characterized and illustrated. Extensive numerical case studies give an overview of the behaviour of stable approximate solutions to the two-dimensional deautoconvolution problem obtained by Tikhonov-type regularization with different penalties and the iteratively regularized Gauss–Newton method.

Key Words.
deautoconvolution, inverse problem, ill–posedness, case studies in 2D, Tikhonov-type regularization, iteratively regularized Gauss–Newton method

AMS Subject Classifications.
47J06, 65R32, 45Q05, 47A52, 65J20

43 The structured distance to singularity of a symmetric tridiagonal Toeplitz matrix.
Silvia Noschese.

Abstract.
This paper is concerned with the distance of a symmetric tridiagonal Toeplitz matrix T to the manifold of similarly structured singular matrices, and with determining the closest matrix to T in this manifold. Explicit formulas are presented, exploiting the analysis of the sensitivity of the spectrum of T with respect to structure-preserving perturbations of its entries.

Key Words.
matrix nearness problem, distance to singularity, eigenvalue conditioning, symmetric tridiagonal Toeplitz structure, structured distance

AMS Subject Classifications.
65F15, 65F35, 15A12, 15A57

60 The bisection eigenvalue method for unitary Hessenberg matrices via their quasiseparable structure.
Yuli Eidelman and Iulian Haimovici.

Abstract.
If N_0 is a normal matrix, then the Hermitian matrices $\frac{1}{2}(N_0 + N_0^*)$ and $\frac{i}{2}(N_0^* - N_0)$ have the same eigenvectors as N_0. Their eigenvalues are the real part and the imaginary part of the eigenvalues of N_0, respectively. If N_0 is unitary, then only the real part of each of its eigenvalues and the sign of the imaginary part is needed to completely determine the eigenvalue, since the sum of the squares of these two parts is known to be equal to 1. Since a unitary upper Hessenberg matrix U has a quasiseparable structure of order one and we express the matrix $A = \frac{1}{2}(U + U^*)$ as quasiseparable matrix of order two , we can find the real part of the eigenvalues and, when needed, a corresponding eigenvector x, by using techniques that have been established in the paper by Eidelman and Haimovici [Oper. Theory Adv. Appl., 271 (2018), pp. 181–200].
We describe here a fast procedure, which takes only 1.7% of the bisection method time, to find the sign of the imaginary part. For instance, in the worst case only, we build one row of the quasiseparable matrix U and multiply it by a known eigenvector

of A, as the main part of the procedure. This case occurs for our algorithm when among the 4 numbers $\pm\cos t \pm i\sin t$ there are exactly 2 eigenvalues and they are opposite, so that we have to distinguish between the case $\lambda, -\lambda$ and the case $\overline{\lambda}, -\overline{\lambda}$. The performance of the developed algorithm is illustrated by a series of numerical tests. The algorithm is more accurate and many times faster (when executed in Matlab) than for general Hermitian matrices of quasiseparable order two, because the action of the quasiseparable generators, which are small matrices in the previous cited paper, can be replaced by scalars, most of them real numbers.

Key Words.
quasiseparable, eigenstructure, Sturm property, bisection, unitary Hessenberg

AMS Subject Classifications.
15A18, 15B10, 15B57, 65F15

89 A note on "Error bounds of Gaussian quadrature formulae with Legendre weight function for analytic integrands" by M. M. Spalević et al..
Aleksandar V. Pejčev.

Abstract.
In paper D. Lj. Đukić, R. M. Mutavdžić Đukić, A. V. Pejčev, and M. M. Spalević, *Error estimates of Gaussian-type quadrature formulae for analytic functions on ellipses – a survey of recent results*, Electron. Trans. Numer. Anal., 53 (2020), pp. 352–382, Lemma 4.1 can be applied to show the asymptotic behaviour of the modulus of the complex kernel in the remainder term of all the quadrature formulas in the recent papers that are concerned with error estimates of Gaussian-type quadrature formulae for analytic functions on ellipses. However, in the paper D. R. Jandrlić, Dj. M. Krtinić, Lj. V. Mihić, A. V. Pejčev, M. M. Spalević, *Error bounds of Gaussian quadrature formulae with Legendre weight function for analytic integrands*, Electron. Trans. Anal. 55 (2022), pp. 424–437, which this note is concerned with, there is a kernel whose numerator contains an infinite series, and in this case the mentioned lemma cannot be applied. This note shows that the modulus of the latter kernel attains its maximum as conjectured in the latter paper.

Key Words.
error bound, quadrature formula, Legendre weight function

AMS Subject Classifications.
65D32, 65D30, 41A55

99 Orthogonality on the semicircle: old and new results.
Gradimir V. Milovanović.

Abstract.
Orthogonal polynomials on the semicircle were introduced by Gautschi and Milovanović in [Rend. Sem. Mat. Univ. Politec. Torino, Special Issue (July 1985), pp. 179-185] and [J. Approx. Theory, 46 (1986), pp. 230-250]. In this paper we give an account of this kind of orthogonality, weighted generalizations mainly oriented to Chebyshev weights of the first and second kinds, including several interesting properties of such polynomials. Moreover, we also present a number of new results including those for Laurent polynomials (rational functions) orthogonal on the semicircle. In particular, we give their recurrence relations and study special cases for the Legendre weight and for the Chebyshev weights of the first and second kind.

Explicit expressions for such orthogonal systems with Chebyshev weights are presented, as well as the corresponding zero distributions.

Key Words.
complex orthogonal systems, recurrence relations, zeros, weight function, orthogonal Laurent polynomials

AMS Subject Classifications.
30C10, 30C15, 33C45, 33C47, 42C05

116 Characterizations of Adjoint Sobolev Embedding Operators with Applications in Inverse Problems.
Simon Hubmer, Ekaterina Sherina and Ronny Ramlau.

Abstract.
We consider the Sobolev embedding operator $E_s : H^s(\Omega) \to L_2(\Omega)$ and its role in the solution of inverse problems. In particular, we collect various properties and investigate different characterizations of its adjoint operator E_s^*, which is a common component in both iterative and variational regularization methods. These include variational representations and connections to boundary value problems, Fourier and wavelet representations, as well as connections to spatial filters. Moreover, we consider characterizations in terms of Fourier series, singular value decompositions, and frame decompositions, as well as representations in finite dimensional settings. While many of these results are already known to researchers from different fields, a detailed and general overview or reference work containing rigorous mathematical proofs is still missing. Hence, in this paper we aim to fill this gap by collecting, introducing, and generalizing a large number of characterizations of E_s^* and discuss their use in regularization methods for solving inverse problems. The resulting compilation can serve both as a reference as well as a useful guide for its efficient numerical implementation in practice.

Key Words.
Sobolev spaces, embedding operators, inverse problems

AMS Subject Classifications.
46N40, 46E35, 47A52, 65J20

145 Optimal averaged Padé-type approximants.
Dusan Lj. Djukić, Rada M. Mutavdžić Djukić, Lothar Reichel, and Miodrag M. Spalević.

Abstract.
Padé-type approximants are rational functions that approximate a given formal power series. Boutry [Numer. Algorithms, 33 (2003), pp 113–122] constructed Padé-type approximants that correspond to the averaged Gauss quadrature rules introduced by Laurie [Math. Comp., 65 (1996), pp. 739–747]. More recently, Spalević [Math. Comp., 76 (2007), pp. 1483–1492] proposed optimal averaged Gauss quadrature rules, that have higher degree of precision than the corresponding averaged Gauss rules, with the same number of nodes. This paper defines Padé-type approximants associated with optimal averaged Gauss rules. Numerical examples illustrate their performance.

Key Words.
Gauss quadrature, averaged Gauss quadrature, optimal averaged Gauss quadrature, Padé-type approximant

AMS Subject Classifications.
65D30, 65D32

Abstract.
This paper presents some preconditioning techniques that enhance the performance of iterative regularization methods applied to image deblurring problems determined by a wide variety of point spread functions (PSFs) and boundary conditions. We first consider the anti-identity preconditioner, which symmetrizes the coefficient matrix associated to problems with zero boundary conditions, allowing the use of MINRES as a regularization method. When considering more sophisticated boundary conditions and strongly nonsymmetric PSFs, we show that the anti-identity preconditioner improves the performance of GMRES. We then consider both stationary and iteration-dependent regularizing circulant preconditioners that, applied in connection with the anti-identity matrix and both standard and flexible Krylov subspaces, speed up the iterations. A theoretical result about the clustering of the eigenvalues of the preconditioned matrices is proved in a special case. Extensive numerical experiments show the effectiveness of the new preconditioning techniques, including when the deblurring of sparse images is considered.

Key Words.
inverse problems, regularization, preconditioning, Toeplitz matrices, Krylov subspace methods

AMS Subject Classifications.
65F08, 65F10, 65F22

Abstract.
In this paper we consider a class of non-standard elliptic transmission problems in disjoint domains.
As a model example, we consider an area consisting of two non-adjacent rectangles. In each subarea, a boundary-value problem of elliptic type is considered, where the interaction between their solutions is described by nonlocal integral conjugation conditions.
An a priori estimate for its weak solution in an appropriate Sobolev-like space is proved. A finite difference scheme approximating this problem is proposed and analyzed. An estimate of the convergence rate, compatible with the smoothness of the input data, up to a slowly increasing logarithmic factor of the mesh size, is obtained.

Key Words.
transmission problem, boundary-value problem, nonlocal boundary condition, finite differences, Sobolev spaces, convergence rate estimates

AMS Subject Classifications.
65N12, 65N15

202 Parameter-free restoration of piecewise smooth images.
Alessandro Lanza, Monica Pragliola, and Fiorella Sgallari.

Abstract.
We propose a novel strategy for the automatic estimation of the two regularization parameters arising in the image decomposition variational model employed for the restoration task when the underlying corrupting noise is known to be additive white Gaussian. In the model of interest, the target image is decomposed in its piecewise constant and smooth components, with a total variation term penalizing the former and a Tikhonov term acting on the latter. The proposed criterion, which relies on the whiteness property of the noise, extends the residual whiteness principle, originally introduced in the case of a single regularization parameter. The structure of the considered decomposition model allows for an efficient estimation of the pair of unknown parameters, that can be automatically adjusted along the iterations with the alternating direction method of multipliers employed for the numerical solution. The proposed multi-parameter residual whiteness principle is tested on different images with different levels of corruption. The performed tests highlight that the whiteness criterion is particularly effective and robust when moving from a single-parameter to a multi-parameter scenario.

Key Words.
image restoration, image decomposition, whiteness principle, ADMM

AMS Subject Classifications.
68U10, 94A08, 65K10.

230 Gauss-type quadrature rules with respect to external zeros of the integrand.
Jelena Tomanović.

Abstract.
In the present paper, we propose a Gauss-type quadrature rule into which the external zeros of the integrand (the zeros of the integrand outside the integration interval) are incorporated. This new formula with n nodes, denoted by $\mathcal{G}_n$, proves to be exact for certain polynomials of degree greater than $2n-1$ (while the Gauss quadrature formula with the same number of nodes is exact for all polynomials of degree less than or equal to $2n-1$). It turns out that $\mathcal{G}_n$ has several good properties: all its nodes are pairwise distinct and belong to the interior of the integration interval, all its weights are positive, it converges, and it is applicable both when the external zeros of the integrand are known exactly and when they are known approximately. In order to economically estimate the error of $\mathcal{G}_n$, we construct its extensions that inherit the n nodes of $\mathcal{G}_n$ and that are analogous to the Gauss-Kronrod, averaged Gauss, and generalized averaged Gauss quadrature rules. Further, we show that $\mathcal{G}_n$ with respect to the pairwise distinct external zeros of the integrand represents a special case of the (slightly modified) Gauss quadrature formula with preassigned nodes. The accuracy of $\mathcal{G}_n$ and its extensions is confirmed by numerical experiments.

Key Words.
Gauss quadrature formula, external zeros of the integrand, modified weight function, quadrature error, convergence of a quadrature formula.

AMS Subject Classifications.
65D30, 65D32, 41A55

Abstract.
This paper describes a method to calculate the potential flow in domains in the complex plane exterior to a finite number of closed curves using conformal mapping. A series method is used to compute the potential flow over multiply connected circle domains. The flow is then mapped from the circle domain to the target physical domain by a method which approximates the Laurent series of the conformal map. The circulations around each boundary can be specified. For flow over multi-element airfoils, the circulations are computed to satisfy the Kutta condition at the trailing edges. The linear systems which are solved on the circle domain for both the potential flow and the conformal maps are of the form identity plus a low-rank matrix, allowing for the efficient use of conjugate-gradient-like methods.

Key Words.
Potential flow, numerical conformal mapping, multiply connected domains, series methods, airfoils, Kutta condition.

AMS Subject Classifications.
30C30, 65E05

Abstract.
The Helmholtz equation is related to seismic exploration, sonar, antennas, and medical imaging applications. It is one of the most challenging problems to solve in terms of accuracy and convergence due to the scalability issues of the numerical solvers. For 3D large-scale applications, high-performance parallel solvers are also needed. In this paper, a matrix-free parallel iterative solver is presented for the three-dimensional (3D) heterogeneous Helmholtz equation. We consider the preconditioned Krylov subspace methods for solving the linear system obtained from finite-difference discretization. The Complex Shifted Laplace Preconditioner (CSLP) is employed since it results in a linear increase in the number of iterations as a function of the wavenumber. The preconditioner is approximately inverted using one parallel 3D multigrid cycle. For parallel computing, the global domain is partitioned blockwise. The matrix-vector multiplication and preconditioning operator are implemented in a matrix-free way instead of constructing large, memory-consuming coefficient matrices. Numerical experiments of 3D model problems demonstrate the robustness and outstanding strong scaling of our matrix-free parallel solution method. Moreover, the weak parallel scalability indicates our approach is suitable for realistic 3D heterogeneous Helmholtz problems with minimized pollution error.

Key Words.
Helmholtz equation, parallel computation, matrix-free, geometric multigrid, preconditioner, scalability

AMS Subject Classifications.
65Y05, 65F08, 35J05

295 Filtered polynomial interpolation for scaling 3D images.
Donatella Occorsio, Giuliana Ramella, and Woula Themistoclakis.

Abstract.
Image scaling methods allow us to obtain a given image at a different, higher (upscaling) or lower (downscaling), resolution to preserve as much as possible the original content and the quality of the image. In this paper, we focus on interpolation methods for scaling three-dimensional grayscale images. Within a unified framework, we introduce two different scaling methods, respectively based on the Lagrange and filtered de la Vallée Poussin type interpolation at the zeros of Chebyshev polynomials of the first kind. In both cases, using a non-standard sampling model, we take (via tensor product) the associated trivariate polynomial interpolating the input image. It represents a continuous approximate 3D image to resample at the desired resolution. Using discrete ℓ^∞ and ℓ^2 norms, we theoretically estimate the error achieved in output, showing how it depends on the error in the input and on the smoothness of the specific image we are processing. Finally, taking the special case of medical images as a case study, we experimentally compare the performances of the proposed methods and with the classical multivariate cubic and Lanczos interpolation methods.

Key Words.
Image resizing, image downscaling, image upscaling, Lagrange interpolation, filtered VP interpolation, de la Vallée Poussin means, Chebyshev nodes

AMS Subject Classifications.
94A08, 68U10, 41A05, 62H35

319 Domain truncation, absorbing boundary conditions, Schur complements, and Padé approximation.
Martin J. Gander, Lukáš Jakabčin, and Michal Outrata.

Abstract.
We show for a model problem that the truncation of an unbounded domain by an artificial Dirichlet boundary condition placed far away from the domain of interest is equivalent to a specific absorbing boundary condition placed closer to the domain of interest. This specific absorbing boundary condition can be implemented as a truncation layer terminated by a Dirichlet condition. We prove that the absorbing boundary condition thus obtained is a spectral Padé approximation about infinity of the transparent boundary condition. We also study numerically two improvements for this boundary condition, the truncation with an artificial Robin condition placed at the end of the truncation layer and a Padé approximation about a different point than infinity. Both of these give new and substantially better results compared to using the artificial Dirichlet boundary condition at the end of the truncation layer. We prove our results in the context of linear algebra, using spectral analysis of finite and infinite Schur complements, which we relate to continued fractions. We illustrate our results with numerical experiments.

Key Words.
domain truncation, ABC, Schur complement, continued fractions, Padé approximants

AMS Subject Classifications.
65N85, 65N06, 65E05, 41A21

Abstract.
We investigate the convergence of difference schemes that approximate the third initial boundary value problem for parabolic equations with time dependent coefficients. An abstract operator method is developed to analyze this equation. An estimate of the rate of the convergence in a special discrete $W_2^{1,1/2}$ Sobolev norm, compatible with the smoothness of the solution is obtained.

Key Words.
parabolic initial boundary value problem, oblique derivative boundary condition, finite differences, Sobolev spaces, convergence rate estimates

AMS Subject Classifications.
65M15